A Preliminaries

We introduced the product Lie group $G := SE_3 \times \mathbb{R}^6 \times (0,1)^{|\Omega|}$ in equation (3). The identity element of the Lie group is $Id = (I_4, 0_6, \frac{1}{2}|\Omega|)$, where $I_4$ denotes the $4 \times 4$ identity matrix. The Lie algebra $\mathfrak{g}$ of $G$ is the product Lie algebra $\mathfrak{g} := se_3 \times \mathbb{R}^6 \times \mathbb{R}^{|\Omega|}$, where $se_3$ is given through

$$se_3 := \left\{ \begin{pmatrix} \omega \times t \\ 0 \\ 1 \end{pmatrix} \mid \omega \in \mathbb{R}^3, t \in \mathbb{R}^3 \right\}. \quad (A.1)$$

$[\omega]_\times$ denotes as usual the operator that maps a vector $\omega \in \mathbb{R}^3$ into the space of skew-symmetric $3 \times 3$ matrices, denoted by $so_3$. We will also write mat$so_3$ for this operation. It is given through

$$\text{mat}_{so_3}(\omega) : \mathbb{R}^3 \rightarrow so_3, \quad \text{mat}_{so_3}(\omega) := \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}. \quad (A.2)$$

To be consistent with the usual inner product, i.e. $\langle \omega, \omega \rangle := \text{tr}([\omega]_\times [\omega]_\times^T)$ we multiply the factor $\sqrt{2}$. The inverse operator of mat$so_3$ is vec$so_3 := (\text{mat}_{so_3})^{-1} : so_3 \rightarrow \mathbb{R}^3$. Given a vector $\omega \in \mathbb{R}^6$ we can map it onto an element in $se_3$ by using the operator mat$se_3 : \mathbb{R}^6 \rightarrow se_3$ that is defined through

$$\text{mat}_{se_3}(\omega) := \left( \text{mat}_{so_3}(\omega_{1:3}) \omega_{4:6} \right). \quad (A.3)$$

Here, $\omega_{i:j}$ denotes the vector with the elements from $i$ to $j$ in $\omega$. The inverse operation is again denoted by vec$se_3 := (\text{mat}_{se_3})^{-1} : se_3 \rightarrow \mathbb{R}^6$. Finally, we define for the Lie algebra $\mathfrak{g}$ the operation vec$\mathfrak{g}$. For $\eta_1 \in \mathbb{R}^6$, $\eta_2 \in \mathbb{R}^6$ and $\eta_3 \in \mathbb{R}^{|\Omega|}$ it is given through

$$\text{mat}_{\mathfrak{g}}(\eta) := \text{mat}_{\mathfrak{g}}((\eta_1, \eta_2, \eta_3)) := (\text{mat}_{se_3}(\eta_1), \eta_2, \eta_3) \in \mathfrak{g}. \quad (A.4)$$

The inverse operation is denote by vec$\mathfrak{g} := \text{mat}_{\mathfrak{g}}^{-1} : \mathfrak{g} \rightarrow \mathbb{R}^{12+|\Omega|}$.

The corresponding Riemannian metric is given for $\eta^i = (\eta^i_E, \eta^i_v, \eta^i_d)$ in $\mathfrak{g}$, $i = 1, 2$ through

$$\langle \eta^1, \eta^2 \rangle_{Id} = \langle \eta^1_E, \eta^2_E \rangle_{se_3} + \langle \eta^1_v, \eta^2_v \rangle + \langle \eta^1_d, \eta^2_d \rangle, \quad (A.5)$$

where $\langle \eta^1_E, \eta^2_E \rangle_{se_3} := \text{tr}((\eta^1_E)^T \eta^2_E)$ is the usual matrix inner product.
A.1 Vectorization of connection functions

Following [1, Section 5.2], we can vectorize the connection function \( \omega \) of the Levi-Civita connection \( \nabla \) for constant \( \eta, \xi \in \mathfrak{g} \) in the following way:

\[
\text{vec}_g(\omega(\eta, \xi)) = \text{vec}_g(\nabla_\eta \xi) = \tilde{\Gamma} \text{vec}_g(\eta) \text{ vec}_g(\xi) ,
\]

(A.6)

where \( \tilde{\Gamma} \) is the matrix whose \((i, j)\) element is the real-valued function

\[
(\tilde{\Gamma}^i_j)_{i,j} := \sum_k (\gamma^k \Gamma^i_{jk}) ,
\]

(A.7)

and \( \Gamma^i_{jk} \) are the Christoffel symbols of the connection function \( \omega \) for a vector \( \gamma \in \mathbb{R}^{12 \times |\Omega|} \). Similarly, permuting indices, we can define the adjoint matrix \( \tilde{\Gamma}^* \) whose \((i, j)\)-th element is given by

\[
(\tilde{\Gamma}^*_{i,j})_{i,j} := \sum_k (\gamma^k \Gamma^i_{kj}) .
\]

(A.8)

This leads to the following equality:

\[
\text{vec}_g(\omega(\eta, \xi)) = \tilde{\Gamma}^* \text{ vec}_g(\eta) \text{ vec}_g(\xi) .
\]

(A.9)

If the expression \( \xi \) in (A.6) is non-constant, we obtain the following vectorization from [1, Eq. (5.7)], for the case of the Lie algebra \( \mathfrak{se}_3 \), i.e.

\[
\text{vec}_{\mathfrak{se}}(\nabla_\eta \xi(x)) = \tilde{\Gamma}_{\mathfrak{se}} \text{ vec}_{\mathfrak{se}}(\eta_x) + D \text{ vec}_{\mathfrak{se}}(\xi_x) \text{ vec}_{\mathfrak{se}}(\eta_x) ,
\]

(A.10)

where \( D \in \mathbb{R}^{6 \times 6} \) can be computed as

\[
(D)_{i,j} = (\text{vec}_{\mathfrak{se}}(D \xi(x)[E'])_{i,j}) , \quad E' = \text{mat}_{\mathfrak{se}}(e^6_j) ,
\]

(A.11)

where \( e^6_j \) denotes the \( j \)-th unit vector in \( \mathbb{R}^6 \).

The adjoint operator \( \text{ad}_{\mathfrak{se}}(\text{mat}_{\mathfrak{se}}(v)) \) can be computed for a vector \( v \in \mathbb{R}^6 \) as follows

\[
\text{vec}_{\mathfrak{se}}(\text{ad}_{\mathfrak{se}}(\text{mat}_{\mathfrak{se}}(v)) \eta) = \text{ad}_{\mathfrak{se}}(\text{mat}_{\mathfrak{se}}(v)) \text{ vec}_{\mathfrak{se}}(\eta) := \left( \begin{array}{cc} \text{mat}_{\mathfrak{se}}(v_{1:3}) & 0_{3 \times 3} \\ \text{mat}_{\mathfrak{se}}(v_{4:6}) & \text{mat}_{\mathfrak{se}}(v_{1:3}) \end{array} \right) \text{ vec}_{\mathfrak{se}}(\eta) ,
\]

(A.12)
\[
\text{ad}^\text{vec}_g(\text{mat}_g(v)) = \begin{pmatrix}
\text{ad}^\text{vec}_{\Omega}(v_{1:6}) & 0_{6 \times 6} & 0_{6 \times |\Omega|} \\
0_{6 \times 6} & 0_{6 \times 6} & 0_{6 \times |\Omega|} \\
0_{|\Omega| \times 6} & 0_{|\Omega| \times 6} & 0_{|\Omega| \times |\Omega|}
\end{pmatrix}.
\]

(A.13)

For the following calculations we require the \textit{symmetry} property of the Levi-Civita connection \(\nabla\) which reads
\[
\nabla_\eta \xi - \nabla_\xi \eta = [\xi, \eta]
\]
where \([\cdot, \cdot] : g \times g \to g\) denotes the Lie bracket on \(g\), cf. [1, p. 97].

A.2 Christoffel symbols on \(\text{SE}_3\)

The Christoffel symbols \(\Gamma^k_{ij}, i, j, k \in \{1, \ldots, 6\}\) for the Riemannian connection on \(\text{SE}_3\) are given by
\[
\begin{align*}
\Gamma^3_{12} &= \Gamma^1_{23} = \Gamma^2_{31} = \frac{1}{2}, \\
\Gamma^2_{13} &= \Gamma^3_{21} = \Gamma^1_{32} = -\frac{1}{2}, \\
\Gamma^6_{15} &= \Gamma^4_{26} = \Gamma^5_{34} = 1, \\
\Gamma^5_{16} &= \Gamma^6_{24} = \Gamma^4_{35} = -1.
\end{align*}
\]
and zero otherwise.

B Proofs of Lemmas and Theorems

\textit{Proof of Lemma 1.} The proof requires to compute the total time derivative of the necessary condition for the optimal state \(x^*\) which is given through
\[
\text{D}_t V(x^*, t) = 0.
\]

(B.1)

The calculation of the time derivative of (B.1) is already given in [2, Eq. (26)–(37)] and results in the following evolution equation:
\[
(x^*(t))^{-1}\dot{x}^*(t) = -\text{D}_2 \mathcal{H}(x^*(t), \mathbf{0}, t) - Z(x^*(t), t)^{-1} \circ (x^*)^{-1}(\text{D}_1 \mathcal{H}(x^*(t), \mathbf{0}, t)).
\]

(B.2)

The derivative of the Hamiltonian regarding the second component is simply
\[
\text{D}_2 \mathcal{H}(x^*, \mathbf{0}, t) = -f(x^*),
\]

(B.3)
such that the evolution equation for the optimal state \(x^*\) reads
\[
(x^*(t))^{-1}\dot{x}^*(t) = f(x^*) - Z(x^*(t), t)^{-1} \circ (x^*)^{-1}(\text{D}_1 \mathcal{H}(x^*(t), \mathbf{0}, t)),
\]

(B.4)

which is Lemma 1.
The calculation of the differential of the Hamiltonian
\[ D_t H(x^*(t), 0, t) \]
is a bit involved but can be calculated component-wise. We will use the shorthands \( I := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in \mathbb{R}^{4 \times 4}, \ e = (0 \ 0 \ 1 \ 0) \in \mathbb{R}^{1 \times 4}, \ g_z = \begin{pmatrix} (\frac{1}{2})d_z(z, t) \end{pmatrix}^{-1} \in \mathbb{R}^4 \) and \( \kappa_z = eEg_z \in \mathbb{R} \). With these expressions we can write the function \( h \) as \( h(z, t) := \kappa_z^{-1}Eg_z \).

We begin with the directional derivative of the Hamiltonian \( H(x, 0, t) \) regarding the camera motion \( E \) for \( x = (E, v, d) \) in a specific direction \( E\eta \in T_E SE_3 \) which is
\[
D_E H(x, 0, t)[E\eta] = \sum_z D_E \phi \left( \frac{1}{2} \|y_z - h_z(x, t)\|_Q \right)[E\eta] \]
\[
= \sum_z \beta \left( \frac{1}{2} \|y_z - h_z(x, t)\|_Q^2 + \nu \right)^{\beta-1} (y_z - h_z(x, t))^\top Q_z(-1) D_E h_z(x, t)[E\eta] \]
\[
= \sum_z \beta (\cdots)^{\beta-1} \left( y_z - h_z(x, t) \right)^\top Q_z(-1) D_E \left( eEg_zIEg_z \right)[E\eta] \]
\[
= \sum_z \beta (\cdots)^{\beta-1}(1) \left( y_z - h_z(x, t) \right)^\top Q_z \left( \kappa_z^{-1}IEg_z - \kappa_z^{-2}eEg_zIEg_z \right) E\eta \]
\[
= \sum_z \beta (\cdots)^{\beta-1}(1) \left( g_z(y_z - h_z(x, t))^\top Q_z \left( \kappa_z^{-1}I - \kappa_z^{-2}IEg_zE \right) E\eta \right) \]
\[
= \sum_z \beta (\cdots)^{\beta-1}(1) \left( g_z(y_z - h_z(x, t))^\top Q_z \left( \kappa_z^{-1}I - \kappa_z^{-2}IEg_z(e)\top - \kappa_z^{-1}I \right) E \right)^\top, \eta \right). \]

From the definition of the Riemannian gradient on \( SE_3 \) follows that it can be computed by the orthogonal projection \( Pr : \mathbb{R}^{4 \times 4} \rightarrow se_3 \) which reads
\[ A \mapsto A \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - A \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]. \quad (B.5)

The resulting Riemannian gradient of the Hamiltonian regarding \( E \) is
\[ D_E H(x, 0, t) = \sum_z \beta (\cdots)^{\beta-1} E Pr \left( (g_z(y_z - h_z(x, t))^\top Q_z \left( \kappa_z^{-2}IEg_z(e)\top - \kappa_z^{-1}I \right) E \right)^\top \]
\[ := EG_E(x) \in T_E SE_3 . \quad (B.6) \]

The gradient of the Hamiltonian can be calculated component-wise, i.e. for each \( z \) in the image domain \( \Omega \) separately. We will use the shorthand \( g_z := - \left( (\frac{1}{2})d_z(z, t) \right)^{-2} \) for the partial derivative \( \frac{\partial}{\partial d_z(z, t)} g_z \).

Then the components of \( D_d H(x, 0, t) \) read
\[ \frac{\partial}{\partial d_z(z, t)} H(x, 0, t) = \frac{\partial}{\partial d_z(z, t)} \sum_{z \in \Omega} \phi \left( \frac{1}{2} \|y_z - h_z(x, t)\|_Q \right) \]
\[
\begin{align*}
\frac{d}{dt} Z(x^*, t) &= \frac{\partial}{\partial d_1(z, t)} \left( \frac{1}{2} \| y_z - h_z(x, t) \|_{Q_z}^2 + \nu \right) + \nu \beta \\
&= \beta(\cdots) \beta^{-1}(y_z - h_z(x, t))^T Q_z(\beta^{-1}) \frac{\partial}{\partial d_1(z, t)} h_z(x, t) \\
&= \beta(\cdots) \beta^{-1}(y_z - h_z(x, t))^T Q_z(\beta^{-1}) \left( -\kappa_z^2 \epsilon E g_z^I E g_z + \kappa_z^{-1} I E g_z^I \right) \\
&= \beta(\cdots) \beta^{-1}(y_z - h_z(x, t))^T Q_z \left( \kappa_z^2 \epsilon E g_z^I E g_z - \kappa_z^{-1} I E g_z^I \right) \\
&= : (G_d(x))_z \in \mathbb{R}. \tag{B.7}
\end{align*}
\]

By stacking these entries to a vector (given a fixed ordering of \( z \in \Omega \), e.g., column-wise), we obtain the expression \( G_d(x) \). Since the Hamiltonian does not depend on the vector \( v \), the corresponding entries are zero such that we finally obtain the gradient of the Hamiltonian regarding \( x = (E, v, d) \) which is

\[
D_1 H(x, 0, t) = T_{1d} L_x (G_E(x), 0_v, G_d(x)) \in T_x \mathcal{G}. \tag{B.8}
\]

**Proof of Lemma 2.** Following Saccon et al. [2] Eq. (51) the evolution equation of the operator \( Z(x^*, t) : \mathfrak{g} \rightarrow \mathfrak{g}^* \) is given through

\[
\begin{align*}
\frac{d}{dt} Z(x^*, t) &= Z(x^*, t) \circ \omega_{(x^*)^{-1}, x^*}, \tag{B.9a} \\
&= Z(x^*, t) \circ \omega_{\mathcal{D}_2 H(x^*, 0, t)} \tag{B.9b} \\
&+ \omega_{(x^*)^{-1}, x^*} \circ Z(x^*, t) \tag{B.9c} \\
&+ Z(x^*, t) \circ \omega_{\mathcal{D}_2 H(x^*, 0, t)} \tag{B.9d} \\
&+ T_{1d} L_x^* \circ \text{Hess}_1 H(x^*, 0, t) \circ T_{1d} L_x^*, \tag{B.9e} \\
&+ T_{1d} L_x^* \circ \mathcal{D}_2 (D_1 H)(x^*, 0, t) \circ Z(x^*, t) \tag{B.9f} \\
&+ Z(x^*, t) \circ \mathcal{D}_1 (D_2 H)(x^*, 0, t) \circ T_{1d} L_x^* \tag{B.9g} \\
&+ Z(x^*, t) \circ \text{Hess}_2 H(x^*, 0, t) \circ Z(x^*, t). \tag{B.9h}
\end{align*}
\]

Since \( Z \in \mathfrak{g}^* \) and \( \mathfrak{g}^* \) is a vector space we can represent \( Z(x^*, t) \) as a matrix \( K(t) \in \mathbb{R}^{(|\Omega|+12) \times (|\Omega|+12)} \) by evaluating \( Z \) for a specific \( \eta \in \mathfrak{g} \) and vectorization, i.e.

\[
\text{vec}_\mathfrak{g}(Z(x^*, t)(\eta)) = K(t) \text{vec}_\mathfrak{g}(\eta). \tag{B.10}
\]

Similarly we can vectorize the full differential equation of \( Z \) to find the evolution equation of the operator \( P(t) \) in Lemma 2.

With \( \text{[B.10]} \) it is obvious to see that the expression in \( \text{[B.9a]} \) can be vectorized as

\[
\text{vec}_\mathfrak{g} \left( \frac{d}{dt} Z(x^*, t)(\eta) \right) = \tilde{K}(t) \text{vec}_\mathfrak{g}(\eta). \tag{B.11}
\]

Second, we consider the expressions in \( \text{[B.9b]} \) and \( \text{[B.9c]} \).

\[
\text{vec}_\mathfrak{g}(Z(x^*, t) \circ \omega_{(x^*)^{-1}, x^*} \circ Z(x^*, t)) = \text{vec}_\mathfrak{g}(\omega_{\mathcal{D}_2 H(x^*, 0, t)}(\eta)).
\]
for the expression in (B.9g). Setting $f$ matrix
\[ \mathbf{C} \]
and $g$ defined in (6). Again, by duality follows
\[ T = K = \mathbf{K} = \mathbf{K} = \mathbf{K} \]

By duality we find that the lines (B.9d) and (B.9e) can be represented as
\[ \text{vec}_g(\omega(x^*)^{-1} - x^*, t)(\eta) + \mathbf{D}_2^\top \mathbf{H}(x^*, t)(\eta) = C_1(x^*, t)\top K(t)\eta. \] (B.12)

The vectorization of (B.9a) can be simply achieved as
\[ \text{vec}_g(Z(x^*, t) \circ \mathbf{D}_1(\mathbf{D}_2\mathbf{H})(x^*, 0, t) \circ T_{\mathbf{L}_x} \eta) \]
\[ = K(t) \text{vec}_g(\mathbf{D}_1(\mathbf{D}_2\mathbf{H})(x^*, 0, t) \circ x^* \eta) \]
\[ = - K(t) \text{vec}_g(\mathbf{D}_1(\mathbf{D}_2\mathbf{H})(x^*, 0, t) \circ x^* \eta) \]
\[ = - K(t) \left( \begin{array}{c} 0_{6 \times 6} \mathbf{I}_6 \mathbf{0}_{6 \times [\mathcal{D}]} \end{array} \right) \text{vec}_g(\eta) \]
\[ = - K(t) C_2(x^*, t) \text{vec}_g(\eta), \]
where $f$ is the function in the paper defined in (6). Again, by duality follows
\[ \text{vec}_g(T_{\mathbf{L}_x} \circ \mathbf{D}_2(\mathbf{D}_1\mathbf{H})(x^*, 0, t) \circ Z(x^*, t) \circ \eta) = C_2(x^*, t)\top K(t) \text{vec}_g(\eta) \] (B.14)

for the expression in (B.9g). Setting $C(x^*, t) := C_2(x^*, t) - C_1(x^*, t)$ gives the matrix $C(x^*, t)$ in Lemma 2.

The expression in (B.9b) can be calculated as
\[ \text{vec}_g(Z(x^*, t) \circ \mathbf{Hess}_2 \mathbf{H}(x^*, 0, t) \circ Z(x^*, t) \circ \eta) \]
\[ = - K(t) \text{vec}_g(\mathbf{Hess}_2 \mathbf{H}(x^*, 0, t) \circ Z(x^*, t)) \] (B.16)
\[ = - K(t) RK(t) \text{vec}_{\mathbf{x}^*}(\eta), \] (B.17)
where $R$ is the weighting matrix in the energy function in the original paper in (8).

It remains to calculate the matrix representation of the Hessian of the Hamiltonian in (B.9f), i.e.

$$
\text{vec}_g(T_{id}L_x^* \circ \text{Hess} \mathcal{H}(x^*, 0, t) \circ T_{id}L_x^* \circ \eta) = : H(x^*, t) \text{vec}_g(\eta). 
$$

(B.18)

The single blocks of the Hessian of the Hamiltonian can be calculated again separately, i.e.

$$
H = \begin{pmatrix}
H_{11} & H_{12} & H_{13} \\
H_{21} & H_{22} & H_{23} \\
H_{31} & H_{32} & H_{33}
\end{pmatrix}.
$$

Note that the entries that address the variable $v$ are all zero since the Hamiltonian does not depend on $v$ for $x = (E, v, d_i)$. Thus, the entries $H_{12}, H_{13}, H_{21}, H_{22}, H_{31}$ are all zero. As we consider the Riemannian Hessian regarding the symmetric Levi-Civita connection, it is sufficient to calculate $H_{11}, H_{33}$ and $H_{13} = H_{31}^\top$.

The matrix $H_{33}$ is a diagonal matrix containing the partial derivatives

$$
\frac{\partial}{\partial d_i(z, t)} (G_{d_i}(x))_z,
$$

(B.19)

where $(G_{d_i}(x))_z$ was calculated in (B.7). The columns of $H_{13}$ can be obtained similarly by calculation of the partial derivatives of the vector representation of the gradient $G_E(x^*)$ in (B.6), i.e.

$$
(H_{13})_{i_z} = \frac{\partial}{\partial d_i(z)} \text{vec}_{se}(\text{Pr}(G_E(x^*))),
$$

(B.20)

where $\bullet$ denotes the full column, and $i_z$ denotes the index of $z$ in a fixed ordering (e.g. column-wise). The matrix $H_{11}$ is a bit more complicated because the Christoffel symbols on $\text{SE}_3$ are not zero. It can be calculated by the general definition of the Riemannian Hessian (cf. [1, Def. 5.5.1]). For $\eta \in \text{SE}_3$ we find the following equalities

$$
\text{vec}_{se}(E^{-1} \text{Hess}_E \mathcal{H}(x^*, 0, t)[E \eta])
\begin{aligned}
\text{Def} & \equiv \text{vec}_{se}(E^{-1} \nabla \eta E_D \mathcal{H}(x^*, 0, t)) \\
\text{Lin} & \equiv \text{vec}_{se}(\nabla \eta E^{-1} D_E \mathcal{H}(x^*, 0, t))
\end{aligned}
$$

(B.20)

The last equation can be evaluated by using the formula in (A.10). For brevity we omit the full calculations.

Thus, evaluation of the differential equation of $Z$ for a $\eta \in \mathfrak{g}$ and vectorization with the operator $\text{vec}_g$ gives us the following equation for $\dot{K}$:

$$
\dot{K}(t) = -K(t)RK(t) - K(t)C(x^*, t) - C(x^*, t)^\top K(t) + H(x^*, t).
$$

(B.21)
We want to avoid the inversion of the operator $K$ in (B.2). Therefore we replace $P(t) := K(t)^{-1}$ in (B.21). By using the well-known rule for the calculation of the differential of an inverse matrix we obtain finally the expression in Lemma 2.

$$
\dot{P}(t) = \frac{d}{dt} K(t)^{-1} = -K(t)^{-1} \dot{K}(t) K(t)^{-1}
$$

$$= -P(t)(-1)K(t)RK(t)P(t) - (-1)P(t)K(t)C(x^*, t)P(t) - (-1)P(t)C(x^*, t)^\top K(t)P(t) - P(t)H(x^*, t)P(t)
$$

$= R + C(x^*, t)P(t) + P(t)C(x^*, t)^\top - P(t)H(x^*, t)P(t)
$ (B.22)

\[\square\]

**Proof of Theorem 1.** By replacing the expression

$$Z(x^*, t)^{-1} \circ \eta = \text{mat}_g(P(t) \text{vec}_g(G(x^*, t)))$$

in (B.2) we obtain the equation (15) in the paper where

$$G(x^*, t) := (G_E(x^*), 0_6, G_d(x^*))$$

denotes the gradient of the Hamiltonian as calculated in (B.8). The initial condition of equation (15) can be found by minimizing the value function (9) for $t = t_0$. Then we find that the optimal initial state is $x^*(t_0) = x_0$. The differential equation for $P$ was already calculated in Lemma 2 in (B.22) and the initial state of $P$ is the inverse of the Hessian of the value function (9) at $t = t_0$ which is $R_0$. This completes the proof. \[\square\]

**References**